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## Implications of Lorentz symmetry violation on a 5D supersymmetric model

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Field models with  $n$  extra spatial dimensions have a larger  $SO(1, 3+n)$  Lorentz symmetry which is broken down to the standard  $SO(1, 3)$  four dimensional one by the compactification process. By considering Lorentz violating operators in a 5D supersymmetric Wess-Zumino model, which otherwise conserve the standard four dimensional Poincare invariance, we show that supersymmetry can be restored upon a simple deformation of the supersymmetric transformations. However, supersymmetry is not preserved in the effective 4D theory that arises after compactification when the 5D Lorentz violating operators do not preserve  $Z_2 : y \rightarrow -y$  bulk parity. Our mechanism unveils a possible connection among Lorentz violation and the Scherk-Schwarz mechanism. We also show that parity preserving models, on the other hand, do provide well defined supersymmetric KK models.

*Keywords:* Supersymmetry; effective model; extra dimensional model; Wess-Zumino model.

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### 1. Introduction

Symmetries play an important role to describe the fundamental interactions on particle physics<sup>1</sup> in the context of the Standard Model (SM). Those can be categorized in two general sets, the so called internal and the space-time symmetries. Internal symmetries do not affect space-time coordinates, but are responsible of implementing the gauge principle in quantum field theories. Space-time symmetries, on the other hand, are useful to label fundamental field properties. In the SM, the internal  $SU(3)_C \times SU(2)_L \times U(1)_Y$  and Lorentz symmetries can not be mixed into a single Lie group, because it is not possible to find a Lie Algebra which contains, in a non trivial way, the generators of both groups. This is possible, however, in supersymmetric theories,<sup>2</sup> which opens the possibility of finding a theory that in-

incorporates the SM interactions and gravity on the same theoretical ground. One of the best candidates for that is String Theory, which needs supersymmetry to be self consistent. As a plus, supersymmetry alleviates some of the theoretical problems of the SM, as the hierarchy problem. On the other hand, String Theory also predicts additional space-like dimensions to the four we see. This last feature had motivated a lot of interest in the past decade for the study of models with extra dimensions, also because they may provide another approach to solve the hierarchy problem.<sup>3,4</sup> Nevertheless, none of these had found so far an experimental confirmation, which indicates that the theory should have a way of hiding the physical implications of both, supersymmetry and extra dimensions. Last is usually explained from the condition that such new dimensions should be rather compact and of very small sizes, whereas supersymmetry should be a broken symmetry at low energy. The actual physical mechanisms beneath those conditions are yet unknown. Exploring the theoretical potential of both ideas still remains as an interesting topic. We will be dealing with it along the present work.

Supersymmetry (SUSY) involves the existence of transformations between bosonic and fermionic fields whose Lie graded algebra in four dimensions is given by

$$[P_\mu, P_\nu] = [P_\mu, Q] = 0, \quad \{Q, \bar{Q}\} \propto 2\gamma^\mu P_\mu, \quad (1)$$

where  $P_\mu$  represents the energy-momentum vector which generates translations in space-time. Also, the  $Q$  spinor generates the supersymmetric transformations and  $\gamma^\mu$  are the Dirac matrices. For space-time dimensions other than four, the Lie graded algebra is preserved, but properties of the spinors and gamma matrices may change.<sup>5</sup>

It is important to remark that the supersymmetric algebra (1) does not involve the generators of the Lorentz group ( $SO(1,n)$ ), which allows to violate Lorentz invariance, at least in an explicit way, while preserving the mathematical form of the Lie graded algebra, that is, preserving SUSY. In Ref. 7, Kostelecky and Berger presented a 4D Wess-Zumino supersymmetric model which also contains specific operators that explicitly break Lorentz invariance.<sup>10</sup> There, they have shown that it is possible to have SUSY in a Lorentz violating theory, provided SUSY transformations between the fields of the model are modified by the simple replacement rule given by

$$p_\mu \gamma^\mu \rightarrow p_\mu (\gamma^\mu + k_{\mu\nu} \gamma^\nu), \quad (2)$$

where  $k_{\mu\nu}$  is a set of parameters which could be seen as the non zero expectation value of a background tensorial field, which, therefore, quantify the violation of the Lorentz symmetry.<sup>11</sup> It is worth noticing that the usual SUSY transformations for the Wess-Zumino model<sup>12</sup> are recovered in the limit  $k_{\mu\nu} \rightarrow 0$ . A number of experiments have been used to impose strong constraints on many possible Lorentz violation parameters. A review with different strong limits can be found in Ref. 13. Notice also that even though the model in Ref. 7 is not considered a free (toy) model, the addition of Lorentz violation operators does not break SUSY, and therefore the

model remains quite unviable to be a good phenomenological model, unless another mechanism for SUSY breaking is claimed.<sup>14</sup>

On the other hand, in the study of field theory models in more than four dimensions, in order to understand why we see only three space-like dimensions, it is usually assumed that all extra dimensions are compactified on a closed space manifold of finite size.<sup>4</sup> However, the compactification process necessarily breaks the extended Lorentz symmetry,  $SO(1, 3 + n)$ , in an implicit way. That is because any compactification has, by definition, the role of introducing a way to distinguish our four space-time dimensions from those extra ones. Therefore, no transformation that mixes normal with extra dimensions may remain as a symmetry after compactification. That is clearly the case, for instance, when compactification on orbifolds is used, where fixed points are introduced, or, either, when fields are located on the so called 4D branes. This implicit rupture of the higher dimensional symmetry is given only over the additional dimensions, because the resulting 4D effective model has the usual 4D Lorentz invariance. In formal terms, the compactification breaks the  $SO(1, 3 + n)$  group into a residual  $SO(1, 3)$  group that only involves transformations among our standard four dimensions, such that Poincare invariance is always preserved at the level of the effective four dimensional theory. On this line of reasoning, it seems interesting to consider the theoretical possibilities of those operators that explicitly break the larger Lorentz invariance, but which at the same time preserve standard 4D Poincare symmetry. The idea has been already explored by Rizzo in Ref. 15, where he has shown how some specific operators in 5D space-time models can achieve an explicit rupture of 5D Lorentz symmetry, but after compactification the effective model remains invariant under the corresponding 4D symmetry. The phenomenological consequences of these toy models were also discussed in there. However, a supersymmetric equivalent of this setup had not been considered so far.

In this work we address two related questions. First of all, we will explore whether or not SUSY can be restored in a 5D field theory where bulk Lorentz invariance is explicitly broken, but where the involved operators do conserve 4D Lorentz symmetry. Second, we will focus on the phenomenological consequences of this approach at the level of the effective (compactified) models. Notice that in some cases it is possible to achieve supersymmetry breaking through the addition of operators over the free model.<sup>6</sup> For these purposes we will follow the 4D proposal on Ref. 7, but here, we will consider a 5D supersymmetric model where some operators that break in an explicit way the  $SO(1, 4)$  Lorentz symmetry had been added. For simplicity, we will consider two explicit extensions of a 5D Wess-Zumino supersymmetric free model which, later, will be compactified using the orbifold  $S^1/Z_2$ . As we will discuss, our results show different phenomenological consequences which depend on the extra dimensional parity assigned to the operators that violate the 5D Lorentz symmetry. Even though the answer to our main question is on the positive at the 5D level, parity violating models turn out to be anomalous. Restoring SUSY in the lasts requires the addition of a 5D term that later generates a mass gap on 4D SUSY partners. This implies the explicit breaking of 4D SUSY, thus, connecting SUSY and Lorentz

symmetry breaking. A close look at this model shows that, on the orbifold, the resulting model appears to be related to the one arising from the Scherk-Schwarz mechanism, where a twist on the boundary conditions is involved.<sup>8,9</sup> This suggests that the last mechanism may have its origin on bulk Lorentz violation.

This work has been organized with the following structure. In the second section we describe both the models and the operators that will be taken into account to achieve the explicit violation of Lorentz symmetry, on a general flat  $5D$  space. The required conditions to preserve  $5D$  SUSY in such models are also discussed there, without regard to the later effects produced by compactification. Section three is dedicated to show the phenomenological consequences on the effective models that result out of the space compactification on the  $S^1/Z_2$  orbifold. Finally, some further discussion and our conclusions are presented in section four. Two appendices have been added to provide our conventions for the construction of the models and the main features of the  $S^1/Z_2$  orbifold, as well as to explain some basic, but required, details on the  $5D$  Wess-Zumino supersymmetric model.

## 2. Five-dimensional supersymmetric models

In the work of Ref. 16, the authors divide the 4D operators that explicitly break Lorentz symmetry in two kinds, those that are CPT invariant, and those that are not. However, as here we are interested only on explicitly breaking the 5D Lorentz symmetry and retrieving the 4D symmetry for the effective models after compactification, there is no need to consider CPT violation (see Ref. 15 for a discussion of CPT in this context). Instead, on 5D, a classification of operators can be made by separating those that preserve parity over the fifth space dimension, from those which do not.

We will start by considering the  $5D$  Wess-Zumino model given by the Lagrangian

$$\mathcal{L}_{WZ} = \partial_M H_i^\dagger \partial^M H_i + i \bar{\Psi} \Gamma^M \partial_M \Psi + F_i^\dagger F_i, \quad (3)$$

where  $H_i$  are two scalar fields,  $i = 1, 2$ ;  $\Psi$  is a Dirac spinor formed by two Weyl spinors, written as  $(\lambda \ \bar{\chi})^T$ , and  $F_i$  are two auxiliary fields (for further details on the model see Appendix A). In above equation  $M$  represents the five-dimensional indices, i.e.  $M = \{\mu, y\}$ , where we use  $\mu = \{0, 1, 2, 3\}$  to refer to the standard four-dimensional indices, whereas the fifth index is denoted as  $y$ . Gamma matrices  $\Gamma^M = \{\gamma^\mu, \gamma^y\}$  are given in chiral representation as

$$\gamma^\mu = \begin{pmatrix} \mathbf{0} & \sigma^\mu \\ \bar{\sigma}^\mu & \mathbf{0} \end{pmatrix}, \quad \gamma^y = i \begin{pmatrix} -\mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}, \quad (4)$$

where  $\sigma^\mu = (\mathbf{1}, \vec{\sigma})$  and  $\bar{\sigma}^\mu = (\mathbf{1}, -\vec{\sigma})$ . Here,  $\vec{\sigma}$  are the usual Pauli sigma matrices and the five-dimensional space-time is assumed to be flat with the Minkowski metric

$$\eta^{MN} = \text{diag}(1, -1, -1, -1, -1). \quad (5)$$

### 2.1. The Lorentz violating operators

Given the fields contained on the Wess-Zumino model, the operators that violate 5D Lorentz symmetry, but preserve 4D Poincare invariance, may only contain scalars ( $H_i$ ) and fermion fields ( $\Psi$ ). As mentioned previously, these operators can be separated in two sets, according to the 5D  $Z_2$  parity. First, the operators that preserve 5D parity, that is, those that are even under the simple mapping  $Z_2 : y \rightarrow -y$ , are given by

$$k_H \partial_y H^\dagger \partial_y H; \quad (6)$$

$$k_\Psi \bar{\Psi} \Gamma^y \partial_y \Psi. \quad (7)$$

On the other hand, there are also two operators where parity over the additional dimension is not preserved, which are given by

$$i\alpha H^\dagger \partial_y H; \quad (8)$$

$$\beta \bar{\Psi} \Gamma^y \Psi. \quad (9)$$

In above,  $k_\Psi$  and  $k_H$  are dimensionless parameters, whereas the parameters  $\alpha$  and  $\beta$  have a mass dimension one. This mass dimensionality shall have important phenomenological consequences as we will seen in detail later. We will discuss both the sets separately, before moving into the corresponding effective 4D theory. It is important to remark that the operators on Eqs. (8) and (9) can be seen as the scalar and fermion fifth current component respectively, and they can interact with a gauge field, but this approach and its consequences after compactification are out of the scope of this work.<sup>17</sup>

### 2.2. Parity preserving case

Let us first consider those operators which respect 5D parity to build an extended Lagrangian,  $\mathcal{L} = \mathcal{L}_{WZ} + \mathcal{L}_L$ , with an explicit rupture of 5D Lorentz symmetry, given by

$$\mathcal{L}_L = k_{H_i} \partial_y H_i^\dagger \partial_y H_i + k_\Psi \bar{\Psi} \Gamma^y \partial_y \Psi. \quad (10)$$

A direct calculation reveals that the model generated with the above Lagrangian is invariant, up to a total derivative, if the following extended supersymmetric transformations are considered

$$\delta_\xi H_i = \sqrt{2} \epsilon_{ij} \bar{\xi}_j \Psi; \quad (11)$$

$$\delta_\xi \Psi = -i\sqrt{2} \epsilon_{ij} (\Gamma^M \partial_M + k_\Psi \Gamma^y \partial_y) \xi_j H_i + \sqrt{2} \xi_i F_i; \quad (12)$$

$$\delta_\xi F_i = -i\sqrt{2} \bar{\xi}_i (\Gamma^M \partial_M + k_\Psi \Gamma^y \partial_y) \Psi, \quad (13)$$

provided that, in order to close the algebra, the following restriction on the coupling parameters is impose,

$$k_{H_i} = k_\Psi^2 + 2k_\Psi. \quad (14)$$

On the other hand, the commutator of two of these so defined SUSY transformations is given by

$$[\delta_\eta, \delta_\xi] \propto 2i\bar{\eta}_i (\Gamma^M \partial_M + k_\Psi \Gamma^y \partial_y) \xi_i, \quad (15)$$

where we can easily see the translational operator dependence and that the supersymmetric transformations for an usual 5D Wess-Zumino model are recovered in the limit  $k_\Psi \rightarrow 0$  (see Appendix A), as it would be expected.

To finalize this subsection, we note the possibility to make a redefinition for the spatial derivative in its fifth component

$$\partial_y \rightarrow (1 + k_\Psi) \partial_y, \quad (16)$$

which allows to rewrite the Lagrangian for this model in the usual superfield formalism as

$$\mathcal{L}_1 = \bar{\Phi}_i \Phi_i|_D + \Phi_1 (1 + k_\Psi) \partial_y \Phi_2|_F + h.c. \quad (17)$$

In this equation,  $\Phi_1$  and  $\Phi_2$  are standard chiral superfields, as described in the Appendix A.

It is worth noticing that the whole effect imposed by the supersymmetry condition (14) is to reduce the overall factor in front of the fifth derivatives to a common one for all field components in the supermultiplet, as indicated by Eq. (17). One may argue that such a constant factor can be removed by a simple rescaling of the fifth coordinate, such that  $y \rightarrow (1 + k_\Psi)y$ . Such a mapping removes the overall factor on the fifth derivatives implied by Eq. (16), rendering an explicitly invariant theory, where Lorentz violating operators had been eliminated, and the extended Lagrangian is reduced to the standard Wess-Zumino model. One should then conclude that supersymmetry does require that the 5D free model be equivalent to a Lorentz invariant one. For a collection of free fields the same conclusion would arise even if the Lorentz violation parameters were non universal. That would be so, since in the absence of interactions, the action part of each free field can be treated separately. Thus, performing the proper mapping in each sector in an totally independent way would be straightforward. Nevertheless, in a realistic model realization of this ideas more fields and interactions must be added. For instance, gauge and other required matter multiplets would be required. In the presence of interactions, as all other required kinetic terms will also involve derivatives on the fifth dimension, these will not allow the complete removal of all Lorentz violating coefficients, unless they were universal. Therefore, Lorentz violation in supersymmetric 5D theories, implemented though parity preserving operators, would become a real effect provided its parameters are non universal. Notice however that maintaining supersymmetry in such interacting models still has to be established. Considering this possibility and the fact that interactions are usually treated in the perturbative regime, we will proceed with our analysis of the effective compactified model in the next section without assuming the rescaling of the fifth coordinate. Nonetheless, we shall still comment on its effects on the Kaluza-Klein particle spectrum.

### 2.3. Parity violating case

Next, let us consider once again the Wess-Zumino model, but now with the addition of an  $\mathcal{L}_\mathcal{L}$  which takes into account the odd parity operators, those where parity over the fifth dimension is violated. Such a Lagrangian can be expressed as

$$\mathcal{L}_\mathcal{L} = i\alpha H_i^\dagger \partial_y H_i + \beta \bar{\Psi} \Gamma^y \Psi + h.c. \quad (18)$$

The associated SUSY transformations for this model are then given by

$$\delta_\xi H_i = \sqrt{2} \epsilon_{ij} \bar{\xi}_j \Psi; \quad (19)$$

$$\delta_\xi \Psi = -i\sqrt{2} \epsilon_{ij} (\Gamma^M \partial_M + i\alpha \Gamma^y) \xi_j H_i + \sqrt{2} \xi_i F_i; \quad (20)$$

$$\delta_\xi F_i = -i\sqrt{2} \bar{\xi}_i (\Gamma^M \partial_M + i\alpha \Gamma^y) \Psi. \quad (21)$$

However, unlike what happens in the previous parity conserving case, here, in order to ensure the invariance under such deformed supersymmetric transformations, the parity violating terms in (18) do require to add a new 5D mass-like scalar term to the Lagrangian, given as

$$\mathcal{L}_m = -\alpha^2 H_i^\dagger H_i. \quad (22)$$

Furthermore, in addition to that, it becomes necessary to impose on the Lorentz violating parameters the restriction  $\beta = -\alpha$ , as it can be easily verified.

For this model, as for the parity preserving case, it is also possible to make a redefinition for the fifth component of the momentum operator, as

$$\partial'_y = \partial_y + i\alpha, \quad (23)$$

in order to simplify the transformations given in Eqs. (19)-(21). Nonetheless, as it is clear, this redefinition does not conserve parity over the fifth dimension ( $\partial'_y \neq \partial'_{-y}$ ). Of course, this was actually expected, since  $\alpha$  is precisely the parameter associated to the parity violating operators in Eq. (18).

The commutator for two of such supersymmetric transformations is now given by

$$[\delta_\eta, \delta_\xi] \propto 2\bar{\eta}_i (\Gamma^M \partial_M + i\alpha \Gamma^y) \xi_i. \quad (24)$$

From here, it is easy to see that for  $\alpha \rightarrow 0$  the same features of the Wess-Zumino SUSY model would be recovered, as expected. However, if  $\alpha \neq 0$  there are important phenomenological implications that shall become explicit once the compactification is realized, as we will see later.

Similarly to the parity conservation model, the redefinition (23) allows to write this model in the superfield formalism through the potentials

$$K(\Phi_i) = \bar{\Phi}_i \Phi_i, \quad (25)$$

$$W(\Phi_1, \Phi_2) = \Phi_1 (\partial_y + i\alpha) \Phi_2. \quad (26)$$

Again, the superpotential shows in an explicit way the Lorentz violation because the superfields  $\Phi_1$  and  $\Phi_2$  do have opposite  $Z_2$  parities and now it is not possible

8 *García-Aguilar J.D., Pérez-Lorenzana A.*

to hide this violation through a redefinition over the  $y$ -coordinate. Therefore, this specific model shows how Lorentz violation and supersymmetry could coexist in the frame of extra dimension models.

Finally, we note that this model can be transformed into the Wess-Zumino model (3) through the next redefinition of the fields

$$\begin{aligned} H_i &\rightarrow e^{-i\alpha y} H_i \\ \Psi &\rightarrow e^{-i\alpha y} \Psi \\ F_i &\rightarrow e^{-i\alpha y} F_i. \end{aligned} \tag{27}$$

Consider for instance the fermion terms, which, under above redefinitions and the SUSY condition on the Lorentz violation parameters, become

$$i\bar{\Psi}\Gamma^M\partial_M\Psi + \beta\bar{\Psi}\Gamma^y\Psi \rightarrow i\bar{\Psi}\Gamma^\mu\partial_\mu\Psi + i\bar{\Psi}\Gamma^y(\partial_y - i\alpha)\Psi + \beta\bar{\Psi}\Gamma^y\Psi = i\bar{\Psi}\Gamma^M\partial_M\Psi.$$

A similar calculation confirms also the case for the scalar and the auxiliary field terms. At a first glance, this observation suggests that the above field transformation can hide the Lorentz violation. This can be understood since the field transformations themselves are not in a  $5D$  Lorentz covariant form, and therefore they do not commute with Lorentz transformations. This leaves an open ambiguity. A SUSY Lorentz violating theory could be regarded as fundamental, which means that it would be derived at such from first principles, from String theory for instance, with the Lorentz violation parameters associated to the vacuum expectation values of some bulk background fields. But then, the fields in the transformed frame where Lorentz violation has been hidden should have odd  $5D$  Lorentz transformations to compensate for the properties of the local phase factor. The opposite situation is also possible. One may introduce an apparent Lorentz violation in a SUSY Lorentz conserving theory through the inverse transformations of those given in Eq. (27), in which case, the effect should be regarded as non physical. To disentangle the ambiguity one would have to relay on the implications that the model has at the effective level, once the  $5D$  theory is sited on a compactified space, as we will discuss below.

### 3. Effective 4D models

As it is usual for extra dimensional models, to get a sensible four dimensional effective theory it is necessary to perform a compactification of the extra space. There are many ways to achieve this. For our following discussion, to be specific, we shall consider a compact fifth dimension on the orbifold  $S^1/Z_2$  of radius  $R$  (see Appendix B for details). We will also assume that the Lorentz violating frame should be regarded as fundamental, and then discuss the implications of the model on the orbifolded theory. That means that, along the analysis, all bulk fields will be regarded to comply with the general periodicity condition  $\phi(y + 2\pi R) = \phi(y)$ .

As it is mentioned in the Appendix B, it is possible to assign a specific Kaluza-Klein (KK) decomposition of the fields in order to make the  $Z_2$  parity explicit. This



can be written for the superfields as

$$\Phi_1 = \frac{1}{\sqrt{\pi R}} \Phi_1^{(0)}(x^\mu) + \sqrt{\frac{2}{\pi R}} \sum_{n=1} \Phi_1^{(n)}(x^\mu) \cos\left(\frac{ny}{R}\right), \quad (28)$$

$$\Phi_2 = \sqrt{\frac{2}{\pi R}} \sum_{n=1} \Phi_2^{(n)}(x^\mu) \sin\left(\frac{ny}{R}\right). \quad (29)$$

It is worth noticing that the chosen parities shall project out half of the KK modes, leaving, however, at each KK level the right configuration of fields as to conform a standard 4D chiral superfield model. This means to say that, in a standard 5D Wess-Zumino theory with the same boundary conditions, the 4D  $N = 2$  SUSY described by the field content will be broken, but full  $N = 1$  SUSY field representations will be preserved by the chosen compactification at each level of the KK tower. Next, we will address the effects introduced in the effective theory by the 5D Lorentz violating operators, that is, for each of the models presented above.

First, for the parity preserving model, the zero mode on the Kaluza-Klein tower contains only the massless fields  $H_{10}$ ,  $\lambda_0$  and  $F_{10}$ . However, the excited modes ( $n \geq 1$ ) are susceptible to the Lorentz violating terms. As it is easy to see, the terms in Eq. (10) become mass terms in the effective 4D theory, and thus, the mass for the  $H_{1n}$ ,  $H_{2n}$  and  $\Psi_n$  fields shall differ from the Lorentz invariant Wess-Zumino model. In this case KK masses are given by

$$m_n^2 = \left(\frac{1 + k_\Psi}{R}\right)^2 n^2, \quad (30)$$

where  $k_\Psi$  can take any real value without creating tachionic states since the minimum value of the function  $k_\Psi^2 + 2k_\Psi$  is minus one. Last spectrum was actually expected due to the redefinition given in Eq. (16) that allows to hide the Lorentz violation terms to have a Lorentz invariant model. Notice that, as one would also expect, the effect of the bulk Lorentz violating terms in the effective theory is indeed equivalent to a rescaling of the size of the compact space. In a model with more than one family, the Lorentz violating parameter  $k_\Psi$  can be family dependent. Therefore, the direct conclusion is that each particle KK tower would have associated to it a different effective size of the compact space. This particle dependent mass gap is a distinguishing feature of the bulk Lorentz violating operators, since in a truly Lorentz invariant theory the mass gap should be universal.

It is also straightforward to check that the  $N = 1$  SUSY remains unbroken. As a matter of fact, upon compactification, our first model can be rewritten in superfield formalism as

$$\begin{aligned} \mathcal{L}_{eff} = & \sum_{n=1}^{\infty} \left[ \Phi_{1n}^\dagger \Phi_{1n} \Big|_D + \frac{n}{R} (1 + k_\Psi) \Phi_{1n} \Phi_{2n} \Big|_F \right] \\ & + \Phi_0^\dagger \Phi_0 \Big|_D + h.c. \end{aligned} \quad (31)$$

where the KK chiral superfields are given by

$$\Phi_0 = H_{10} + \sqrt{2}\lambda_0 + \theta^2 F_{10}; \quad (32)$$

$$\Phi_{1n} = H_{1n} + \sqrt{2}\lambda_n + \theta^2 F_{1n}; \quad (33)$$

$$\Phi_{2n} = H_{2n} + \sqrt{2}\chi_n + \theta^2 F_{2n}. \quad (34)$$

On the other hand, in our second model, the parity violating case, compactification leads to a great difference relative to the first model, because the terms which violate parity on the extra dimension, in Eq. (18), do not have any contribution on the effective model, since being odd their integral over the extra dimension simply vanishes. However, that is not the case for the additional term in Eq. (22). At the level of the effective model, as it is easy to see, this term becomes an universal mass term for all scalar KK modes. This is particularly interesting for the zero mode level theory, because it means a mass term for the scalar field which has no equivalent for the fermion field. Thus, we get the zero mode spectrum

$$m_{H_{10}}^2 = \alpha^2; \quad (35)$$

$$m_{\lambda_0}^2 = 0. \quad (36)$$

This, of course, represents a mass gap that evidences SUSY breaking on the  $4D$  effective theory. It is worth stressing that the mass-like term in Eq. (22) was required to restore SUSY at the five dimensional level, due to the explicit breaking of  $5D$  Lorentz invariance. Nonetheless, it is precisely this same term which breaks SUSY in the effective KK theory, where  $4D$  Lorentz invariance holds. Interestingly enough, the term in consideration is precisely what is called a soft-breaking term. Furthermore, we notice that the emergence of the mass gap is actually independent of the chosen compactification, since there is no term in the Lagrangian that may generate an equivalent mass for the zero mode fermions.

A mass gap appears of course in the same way for all the excited modes, for which we get

$$m_{H_{1n}}^2 = m_{H_{2n}}^2 = \alpha^2 + \frac{n^2}{R^2}; \quad (37)$$

$$m_{\lambda_n}^2 = m_{\chi_n}^2 = \frac{n^2}{R^2}. \quad (38)$$

This result shows that SUSY is broken in all the levels of the  $4D$  effective theory. Although this is a tree level calculation, at least for this toy model the mass gap is valid at any order, since we have not consider interactions so far, which could incorporate non universal mass radiative corrections. Also, we notice that there is no restriction to the value for the  $\alpha$  parameter in this case, besides that it should be a real number.

By looking at the mass spectrum, we notice that this resembles the mass structure of KK towers in  $5D$  models where supersymmetry is broken by the Scherk-Schwarz (SS) mechanism.<sup>8,9</sup> The SS mechanism breaks supersymmetry by imposing on the bulk fields the non trivial periodic condition  $\phi(y + 2\pi R) = e^{i2\pi q T} \phi(y)$ ,

where  $T$  must be a non trivial generator of a global symmetry of the  $5D$  theory and  $q$  its associated charge. Notice, however, that the mass spectrum in Eq. (38) is shifted by an universal radius independent term, whereas in the former the twist usually appears as a shift on the KK index, of the form  $n \rightarrow n + q$ . The twisted condition implies that the fields can not longer be expressed in the standard KK mode expansion described by Eq. (29). Instead, the twisted fields are expressed as

$$\phi(y) = e^{iqTy/R} \tilde{\phi}(y) , \quad (39)$$

where  $\tilde{\phi}$  stands for a new field with standard periodicity conditions,  $\tilde{\phi}(y + 2\pi R) = \tilde{\phi}(y)$ . Obviously,  $\tilde{\phi}$  does expand in the usual KK modes, according with its own parity. When building models such a twist is usually assumed ad hoc. No further physical reason is claimed beneath the mechanism. Interestingly enough, the twisted transformation (39) is actually equivalent to the ones given in Eq. (27) that are used to hide Lorentz violation in the parity violating model. Furthermore, by considering our previous discussion, where we assumed that the fields in the Lorentz violating mode have standard periodicity conditions, it is straightforward to see that the transformation (27) actually maps our model into a frame where the SS boundary conditions hold for all fields, but where the  $T$  generator is just the identity with an universal charge identified as  $q = \alpha R$ . This is of course distinctive from the standar implementation of the SS mechanism. Therefore, we can argue that our model actually incorporates a SS-like twist in a natural way, and that the last has a very well identified origin on the  $5D$  supersymmetric Lorentz violation produced by non parity conserving operators. This results may unveil a deeper connection among fundamental Lorentz violation in supersymmetric higher dimensional theories and the SS mechanism.

#### 4. Concluding remarks

In this work we have presented a study of an extended Wess-Zumino supersymmetric model on five space-time dimensions, where  $5D$  Lorentz symmetry is violated in an explicit way. All possible field operators, up to mass dimension five, with such an explicit violation had been considered. These operators can be classified as even or odd under the  $Z_2$  fifth dimensional parity. Thus, we have considered the two most general extended models: the one where  $Z_2$  parity is conserved and the one where parity over the additional dimension is not present. It has been argue that, in both the cases, supersymmetry does still remain as a symmetry of the  $5D$  theory under a set of deformed field transformations, provided some specific conditions on the Lorentz violation parameters are met. However, for the non parity preserving models, restoring supersymmetry does also require the addition of an universal scalar mass term. It is worth mentioning that the SUSY transformations for both the models can be rewritten as those of a supersymmetric Wess-Zumino model through an ad hoc redefinition on the fifth momentum component.

In both the cases, the Lorentz violating terms can be hidden through some appropriate transformations, making the theory to appear as a Lorentz invariant one.

Either by scaling the extra dimension, for the parity even theory, or by performing a non Lorentz covariant transformation in the parity odd model. However, since any extra dimensional theory should consider the extra space to be compact, the effects of such transformations are reflected as a deformation of the boundary conditions. Due to this, it seems likely that a simpler interpretation and treatment of the effective model would arise in the Lorentz violating frame.

As a matter of fact, after compactification is incorporated in the parity preserving model, the only indication about the existence of the  $5D$  Lorentz violating terms is a shifting of the squared masses for all KK excited modes. The shifting comes out to be proportional to the squared KK number, and it is consistent with the coordinate rescaling that hides the Lorentz violating terms in the bulk. The rescaling is in general expected to be particle dependent, and thus, different particles will show different shifting. This is a distinctive feature of the model with respect to the truly Lorentz invariant free theories, where the KK spectrum is expected to be universal. The zero mode fields, however, do remain massless. In this case,  $N = 1$  SUSY is preserved after compactification on the  $S^1/Z_2$  orbifold, and thus, it is possible to rewrite the effective  $4D$  theory in terms of an infinity tower of chiral superfields. Here a comment is in order. Along our discussion we have only analyzed a free particle model. The introduction of interactions in the presence of  $5D$  SUSY and Lorentz violation is an issue that is pending to be analyzed. The restoration of SUSY may be troublesome, since in the rescaled frame, where Lorentz violation is hidden, different fields would appear to have different periodicity, and that could make difficult for SUSY to prevail at the effective theory, which, yet, can be a desirable feature.

On the other hand, the non  $Z_2$  parity preserving model turns out to be the most interesting, at least theoretically. The additional scalar mass term required by the deformed SUSY, after compactification, becomes a supersymmetric soft breaking term for the KK tower that affects even the zero mode. Interestingly enough, the effective  $4D$  theory of this model turns out to be non supersymmetric, even though its  $5D$  parent it is so. Universality, and non derivative nature of the mentioned mass term, indicates that this results is independent of the choice for the compact space. Another point to remark is that the compactification process over the  $S^1/Z_2$  orbifold, that we have considered for our analysis, cancels out both the operators which violate  $5D$  Lorentz symmetry. And so, these operators do not have any implication whatsoever in the effective four dimensional model. They are canceled out after integrating over the extra dimension due to their explicit violation of  $Z_2$  parity.

We have shown that the transformation that may be used to hide the Lorentz violating operators in the bulk do actually transforms the fields into some with similar properties as those required by the Scherk-Schwarz mechanism. In this last frame, the field boundary conditions acquire a natural twist, with a charge proportional to the Lorentz violating parameter of the theory. That may explain the breaking of SUSY on the effective  $4D$  theory, but also suggest that bulk Lorentz violation may be the physics beneath the SS mechanism. Nonetheless, in this realization of

the SS mechanism, the KK spectrum is quite different to the one usually obtained when non trivial symmetries are used (see for instance ref. 9). We think this idea does deserve further study.

Finally, the results we have presented may also suggest a link for susy soft breaking terms to parity and Lorentz invariance violation in 5D models. The realization of the this idea on more realistic models may deserve further attention too. Extending our results by taking into account interaction between fields, either by adding them in the superpotential or by the introduction of gauge fields, which would make the model more realistic, seems as an interesting possibility. In particular, it would be quite interesting to address the question of a possible connection among the parity violating operators and the soft breaking terms needed in any realistic supersymmetric model, as in the MSSM.

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### Appendix A. 5D supersymmetric Wess-Zumino model

The first supersymmetric field theory in the context of 4D was presented by Wess and Zumino.<sup>12</sup> It is straightforward to promote such model to 5D. For that, we just have to take into account two scalar fields,  $H_i$ , a Dirac spinor,  $\Psi$  and two auxiliary fields,  $F_i$ , for  $i = 1, 2$ . The Lagrangian for such a model is then given by

$$\mathcal{L}_{WZ} = \partial_M H_i^\dagger \partial^M H_i + i \bar{\Psi} \Gamma^M \partial_M \Psi + F_i^\dagger F_i, \quad (\text{A.1})$$

where  $M = 0, 1, 2, 3, 5$ . It is easy to see that this last Lagrangian is invariant, up to a total derivative, under the following SUSY transformations

$$\delta_\xi H_i = \sqrt{2} \epsilon_{ij} \bar{\xi}_j \Psi; \quad (\text{A.2})$$

$$\delta_\xi \Psi = -i \sqrt{2} \epsilon_{ij} \Gamma^M \partial_M \xi_j H_i + \sqrt{2} \xi_i F_i; \quad (\text{A.3})$$

$$\delta_\xi F_i = -i \sqrt{2} \bar{\xi}_i \Gamma^M \partial_M \Psi. \quad (\text{A.4})$$

Here,  $\epsilon_{ij}$  is the antisymmetric tensor ( $\epsilon_{21} = 1 = -\epsilon_{12}$ ) and  $\xi_i$  is a symplectic Majorana spinor which represents a constant parameter for the supersymmetric transformations.

As a remainder to the reader, a symplectic Majorana spinor satisfy the relation  $\xi_i = \epsilon_{ij} C \bar{\xi}_j^T$ , where  $C$  is the charge conjugation operator. They can be written through two Weyl spinors as

$$\xi_1 = \begin{pmatrix} \varepsilon \\ \bar{\eta} \end{pmatrix}; \quad \xi_2 = \begin{pmatrix} \eta \\ -\bar{\varepsilon} \end{pmatrix}. \quad (\text{A.5})$$

It is thanks to these spinors that it is possible to write the superalgebra as

$$\{Q_i, Q_j\} = 2\Gamma^M P_M \delta_{ij}. \quad (\text{A.6})$$

Similar to the 4D case, the 5D Wess-Zumino model can be written in standard superfield notation,<sup>18</sup> by considering the following two chiral superfields

$$\Phi_1 = H_1 + \sqrt{2}\theta\lambda + \theta^2 F_1; \quad (\text{A.7})$$

$$\Phi_2 = H_2 + \sqrt{2}\theta\chi + \theta^2 F_2, \quad (\text{A.8})$$

in terms of which the action for the Wess-Zumino Lagrangian can be written as

$$S_{WZ}^5 = \int d^4x dy \left[ \int d^4\theta \left( \Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2 \right) + \int d^2\theta \Phi_1 \partial_y \Phi_2 + h.c. \right] \quad (\text{A.9})$$

assuming  $\Psi = (\lambda, \bar{\chi})^T$ .

## Appendix B. The compactification and the orbifold $S^1/Z_2$

Effective 4D theory arises from an extra dimensional space-time model when a compactification of the extra dimensions is imposed. This compactification process dictates many of the geometric properties of the effective model and, in many cases, it has an explicit influence over the couplings between the effective fields.

In 5D models the most easy way to achieve this compactification process involves to consider the extra dimension to be a circle of radius  $R$ . This selection allow us to give a explicit description to the fields in terms of a Fourier expansion,

$$\phi(x^\mu, y) \sim \sum_n \phi_n(x^\mu) e^{i \frac{n}{R} y}. \quad (\text{B.1})$$

where the  $n$ -th mode,  $\phi_n$ , corresponds to one of the cyclic modes that moves around the extra dimension. As an example, consider a free scalar field,  $\phi$ , whose 5D Lagrangian is given by

$$\mathcal{L} = \partial_M \phi^\dagger \partial^M \phi. \quad (\text{B.2})$$

The compactification on the circle then leads to the effective 4D model described by the Lagrangian

$$\mathcal{L}_{eff} = \int_{-\pi R}^{\pi R} \partial_M \phi^\dagger \partial^M \phi dy \quad (\text{B.3})$$

$$= \sum_{n=0}^{\infty} \left( \partial_\mu \phi_n^\dagger \partial^\mu \phi_n - \frac{n^2}{R^2} |\phi_n|^2 \right), \quad (\text{B.4})$$

where we can see that the compactification provides, from a purely 5D massless model, an infinite set of scalar Kaluza-Klein field modes, where only one of them, the zero mode, remains massless.

Compactification on a circle, however, does not assign explicit extra dimensional parity eigenvalues to the fields. This can be done, if the discrete group  $Z_2$  over the circle is included. This amounts to include an additional identification of opposite points along the circle, those associated by the transformation  $Z_2 : y \rightarrow -y$ . With

this, it becomes now possible to label the fields with the parity eigenvalues  $\pm 1$ . That means to consider effective field expansions in the two following possible forms

$$\phi_+(x^\mu, y) \sim \sum_n \phi_n(x^\mu) \cos\left(\frac{n}{R}y\right), \quad (\text{B.5})$$

$$\phi_-(x^\mu, y) \sim \sum_n \phi_n(x^\mu) \sin\left(\frac{n}{R}y\right). \quad (\text{B.6})$$

This last decomposition for the fields is the right expansion to consider when compactification is done over the  $S^1/Z_2$  orbifold. Notice that the effect is the absence of half of the KK modes with respect to (B.1).

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